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3 (Sem–5/CBCS) STA HE 1

2021

(Held in 2022)

STATISTICS

(Honours Elective)

Paper : STA–HE–5016

(Operations Research)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed: 1×7=7

   (a) Operations research came into existence

   (i) in the year 1940

   (ii) in the military context

   (iii) during World War I

   (iv) during World War II

   (Choose the correct option)

   (b) Define slack and surplus variables.

Contd.

(c) Which of the following is not correct with LPP ?

(i) Proportionality

(ii) Uncertainty

(iii) Additivity

(iv) Divisibility

(Choose the correct option)

(d) The solution to a transportation problem with m sources and n destinations is feasible, if the number of allocations are

(i) $m+n-1$

(ii) $m+n+1$

(iii) $m+n$

(iv) $m \times n$

(Choose the correct option)

(e) Economic order quantity (EOQ) results in

(i) equalisation of carrying cost and procurement cost

(ii) minimization of setup cost

(iii) reduced chances of stockouts

(iv) favourable procurement price

(Choose the correct option)

(f) What is lead time ?

(g) What is two-person zero-sum game ?

2. Answer the following questions : 2×4=8

(a) Explain the graphical method of solving an LPP involving two variables.

(b) Define —

(i) Basic variables;

(ii) Basic feasible solution;

(iii) Degenerate basic feasible solution.

(c) An oil engine manufacturer purchases lubricants at the rate of Rs. 42 per piece from a vendor. The requirement of these lubricants is 1,800 per year. What should be the order quantity per order, if the cost per placement of an order is Rs. 16 and inventory carrying charge per rupee per year is only 20 paise ?

(d) Define saddle point. Is it necessary that a game should always possess a saddle point ?

3. Answer **any three** of the following questions : 5×3=15

(a) A home decorator manufactures two types of lamps, say *A* and *B*. Both lamps go through two technicians — first a cutter, second a finisher.

Lamp *A* requires 2 hours of the cutter's time and 1 hour of the finisher's time. Lamp *B* requires 1 hour of the cutter's and 2 hours of the finisher's time. The cutter has 104 hours and finisher 76 hours of available time each month. Profit on one lamp A is Rs. 6.00 and on one B lamp is Rs. 11.00. Assuming that he can sell all that he produces, how many of each type of lamps should he manufacture to obtain the best return ? Formulate the problem as LPP.

(b) Explain transportation problem and show that it can be considered as an LPP.

(c) Find the basic solutions of the set of equations

$$2x_1 + 4x_2 - 2x_3 = 10$$
$$10x_1 + 3x_2 + 7x_3 = 33$$

(d) Explain north-west corner rule for finding an initial basic feasible solution for a transportation problem.

(e) In a game of matching coins with two players, suppose A wins one unit of value when there are two heads, wins nothing when there are two tails and losses ½ unit of value when there are one head and one tail. Determine the pay-off matrix, the best strategies for each player and the value of the game to A.

Answer the following questions : 10×3=30

4. (a) (i) Prove that, if X_B is a basic feasible solution of the LPP : Max, $Z = CX$ such that $AX = b$, $x \geq 0$ such that for this all $Z_j - C_j \geq 0$ for all non-basic variables, them X_B must be maximum feasible solution, where $Z_j = C_B B^{-1} a_j$. 5

(ii) Solve graphically the following LPP:

Maximize $Z = 2x_1 + 3x_2$

subject to

$$x_1 + x_2 \leq 1$$
$$3x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

5

Or

(b) Solve the LPP by simplex method :

Maximize $Z = 5x_1 + 3x_2$

subject to

$$3x_1 + 5x_2 \leq 15$$
$$5x_1 + 2x_2 \leq 10$$
$$x_1, x_2 \geq 0$$

10

5. (a) (i) Prove that a necessary and sufficient condition for the existence of a feasible solution to an $m \times n$ transportation problem is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ where a_i and b_j denote the availability and requirement at ith origin and jth destination respectively. 5

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(ii) Obtain an initial basic feasible solution to the following transportation problem using the north-west corner rule : 5

		Desination				
		D	E	F	G	Availability
	A	11	13	17	14	250
Origin	B	16	18	14	10	300
	C	21	24	13	19	400
Requirement		220	225	275	250	

Or

(b) (i) Discuss least cost method of solving the transportation problem. 4

(ii) What are the costs associated with inventory ? Distinguish between deterministic and stochastic models in inventory theory. 6

5. (a) (i) Show that the maximin value of a game is less than or equal to the minimax value of the same game. 4

(ii) Solve the game whose pay-off matrix is given by

		B			
		I	II	III	IV
A	I	3	2	4	0
	II	2	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

6

Or

(b) Prove that in the inventory problem of economic lot size with uniform demand and unequal times of production run, the optimal lot size for each production run is given by $Q^0 = \sqrt{\frac{2DC_0}{C_1}}$ and the optimal total cost (TC_0) is given by $TC_0 = \sqrt{2DC_0C_1}$, all the symbols have their usual meaning. 10

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